

# Model of Unsteady Aerodynamic Coefficients of a Delta Wing Aircraft at High Angles of Attack

L. Planckaert

ONERA

System Control and Flight Dynamics Department  
5, Boulevard Paul Painlevé  
59045 Lille Cedex, France

## ABSTRACT

Several approaches for modelling the longitudinal aerodynamic coefficients of a fighter aircraft at high angles of attack, including the unsteady effects are presented. A traditional approach where the model of the coefficient arises in the form of the steady and unsteady effects, the unsteady effect being modelled by means of transfer functions. The second approach uses an internal variable descriptive of the flow field: the vortical state of the flow on the wing. An example of application of this method through neural network model is described.

## NOMENCLATURE

$\alpha$	angle of attack
$\beta$	side slip angle
$\delta$	control surface position
$\Lambda$	wing sweep angle
$\Omega$	rotation rate
$p, q, r$	roll rate, pitch rate, yaw rate
$x$	vortex burst location
$C_N$	normal force coefficient
$C_L$	lift coefficient
$C_m$	pitch moment coefficient
$s$	Laplace variable

## 1. INTRODUCTION

The interest of the near-stall or post-stall flight is multiple. The studies on this subject have a direct impact on the evaluation of the safety of the flight, on the performances in terms of landing distance, and on the increase in the manoeuvrability of the fighters due to the high lift values in this domain, which potentially gives a tactical advantage in air combat.

Moreover, since years, new flow control concepts have been studied and some of them have shown their ability to overcome difficulties (loss of control in yaw, asymmetry of forebody, roll instabilities ) of the flight at high angles of attack and make this flight domain more attractive. At high angles of attack, unsteady aerodynamics has to be taken into account since it can reach up to 30% of the maximum aerodynamic lift and can induce strong changes in the stability of the flight. This is why, it is necessary to have a precise model of aerodynamic forces and moment, to be able to design efficient control laws or to evaluate the capabilities of a fighter in term of maneuverability.

To develop such a model, a specific experimental data set is necessary. These relevant data are obtained thanks some dynamic rigs.

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## 2. TEST FACILITIES

ONERA disposes of two test facilities to study unsteady aerodynamics at high angles of attack:

### 2.1 The rotary balance

The rotary balance (figure 1) is settled in the vertical wind tunnel of ONERA-Lille. Two types of dynamic motion are carried out :

- coning motion :

$$\alpha = \alpha_c \quad \beta = \beta_c \quad \text{with following kinematic relationship : } p = \Omega \cos \alpha_c \cos \beta_c, q = \Omega \sin \beta_c, r = \Omega \sin \alpha_c \cos \beta_c$$

- oscillatory motion :

$$\alpha = \alpha_c + \Delta \alpha \sin(\Omega t), \beta = \beta_c + \Delta \alpha \cos(\Omega t) \quad \text{with } p = \Omega \cos \alpha_c \cos \beta_c, q = \Omega \sin \beta_c, r = \Omega \sin \alpha_c \cos \beta_c$$

maximum wind velocity is 40m/s. The maximum rotation rate is  $\Omega=600^\circ/\text{s}$ . The maximum amplitude of the angle of attack during the test is  $\Delta \alpha_{\max}=20^\circ$ .

### 2.2 "PQR" apparatus

The "PQR" apparatus (figure 2) is settled in the horizontal wind tunnel of ONERA-Lille. This apparatus enables to carry out movements respecting the relation  $q=\dot{\alpha}$ . The tests usually made are oscillatory motion of angle of attack and movements at constant pitch rate. Pitch acceleration is limited to  $10000^\circ/\text{s}^2$ , for a mass model lighter than 6Kg. The maximum wind velocity is 40m/s.

## 3. MODEL BY MEANS OF TRANFERT FUNCTION

In the field of flight dynamics, various methods are used to model the aerodynamic coefficients. Most current methods are based on a development in Taylor series. The aerodynamic coefficients then appeared as a sum of linear terms depending on the inputs of the system. Thus, a longitudinal aerodynamic coefficient will arise in the following form:  $C = C_0(\alpha, \beta) + C_q q + C_{\dot{\alpha}} \dot{\alpha} + C_{\delta} \delta$ . This type of representation is well suited to the flight at low angles of attack but cannot be applied in the field where the streamline flow is separated or when vortex burst phenomena exist, because in this case the representation of the system is strongly non linear and the dynamic derivatives may depend on the amplitude and on the frequency of the motion. The representation of Taylor can be generalised by using a formulation with indicial responses and using the principle of superposition. The coefficient C can then be written in the following form

$$C = \sum_e \int_0^t h_e(t-\tau) e(\tau) d\tau \quad \text{where } e \text{ represents the descriptive inputs of the considered coefficient (angle of attack, side slip angle, pitch rate, roll rate, yaw rate, control surface setting...)}$$

An alternative of this method has been used for several years at ONERA/DCSD/Lille to model the unsteady effects at high angles of attack. The longitudinal aerodynamic coefficient arises then in the form  $C = C_0(\alpha, \beta) + C_q(\alpha)q + C_{\Omega}|\Omega| + C_{dyn} + C_{\delta}\delta$  where  $C_{dyn}$  is solution of the differential equation  $\tau \dot{C}_{dyn} + C_{dyn} = k \dot{\alpha}$  ( $k$  and  $\tau$  are functions of the angle of attack, of its time derivative...). The coefficients  $k$  and  $\tau$  are identified by means of a least squares method in a vicinity of the considered point. The data comes from a set of tests where the angle of attack varies in a sinusoidal way, other inputs remaining constant. The other terms of the development in Taylor series of the coefficient are obtained using specific tests. Making the time constant  $\tau$  a function of the time derivative of the incidence enable this parameters to depend on the direction of the motion ( $\tau$  is smaller when the angle attack is increasing as a result of the separation of the streamline flow, of the propagation of the vortex burst point than when the angle of attack is decreasing as a consequence of the reattachment of the flow). However this structure of model of "black box" type requires a significant number of tests to produce a representative model. Moreover, difficulties can appear in modelling some configurations of modern combat aircraft.

Another alternative was developed to overcome this difficulty and to get more quickly a model in a more restricted domain of variation. It turns out that the differential equation  $\frac{C_{dyn}}{\dot{\alpha}} = k(\alpha) + \frac{g(\alpha)}{1 + \tau(\alpha)s}$  is well suited to describe the time evolution of  $C_{dyn}$ . The parameters  $k, g$  and  $\tau$  are identified at various angles of attack using aerodynamic tests on the rotary balance. At each angle of attack, several tests (at least two) are carried out at various frequencies  $f_j$  with  $\alpha = \alpha_k + \Delta\alpha \sin(2\pi f_j t)$ . The terms relative to the static effects are extracted from tests at very low frequency, those relative to the rotation rate from tests at constant rotation rate and angle of attack. It is then possible to isolate the term  $C_{dyn}$ . If only the first harmonic term in the development in Fourier series of  $C_{dyn}$  is retained, the following relation is obtained:

$$C_{dyn} = F_{k,j} \Delta\alpha e^{2i\pi f_j t} = \left[ k(\alpha_k) + \frac{g(\alpha_k)}{1 + \tau(\alpha_k) 2i\pi f_j} \right] 2i\pi f_j \Delta\alpha e^{2i\pi f_j t}.$$

The parameters  $k, g$  and  $\tau$  are obtained at each angle of attack by minimising the criterion

$$J_k = \sum_j \left| F_{j,k} - \left[ k(\alpha_k) + \frac{g(\alpha_k)}{1 + \tau(\alpha_k) 2i\pi f_j} \right] 2i\pi f_j \right|^2$$

with respect to these parameters. It can be noted that the parameter  $\tau$  varies little with the angle of attack from the pre stall to the stall domain and has a typical reduced value of about 10. The evolution of the parameters  $k, g$  with the angle of attack is presented figure 3. It can be noticed that  $|k| \ll |g|$  in the case of CN. Figure 4 presents the comparison between model and test for reduced pulsation of 0.18, 0.12 and 0.06 and shows the good behaviour of this model. However, the extrapolation of this model for large amplitude tests does not give satisfactory results.

#### 4. STATE REPRESENTATION INTEGRATING THE PHYSICS OF THE FLOW

At the beginning of the 90s, Goman and Khrabrov [ 1 ] have used the suction analogy of Polhamus [ 2], in order to obtain a model of CL including the effect of vortex burst phenomenon on a delta wing with large sweep angle. The CL expression is written as :

$CL = k_p \sin \alpha \cos^2 \alpha + x^2 k_v \sin^2 \alpha \cos \alpha$  where  $x$  is the location of the vortex burst point location reduced by the aerodynamic chord. Unsteady effect at high angles of attack is caused by the movement of the vortex burst whose location is solution of the differential equation  $\tau_1 \dot{x} + x = x_0(\alpha - \tau_2 \dot{\alpha})$ . In this expression,  $x_0(\alpha)$  is the location of the vortex burst point in static conditions. The authors in addition propose a more general representation for the delta wing in the form  $CL = CL(\alpha, x) + CL_q q + CL_{\dot{\alpha}} \dot{\alpha}$ . Works of Huang and Hanff [ 3 ] use a similar formulation of CL by using the analogy of suction of Polhamus with a corrective term issued from experimental pressure data, the transfer function relating the vortex burst point to the angle of attack has a more complex form. In [ 5 ], Greenwell and Wood show that the position of the vortex burst point is governed by a second-order transfer function whose input is the angle of attack.

These results enable to consider a representation of the longitudinal aerodynamic coefficients in the subsonic flight envelope, with good predictive capabilities by using an internal variable of the flow, in fact the location of the vortex burst point. However, it remains delicate to determine experimentally the position of the vortex burst point in the case of moderate sweep angle delta wing with leading edge discontinuities (slats...). Moreover, the formal expression of the term  $C(\alpha, x)$  is very difficult to obtain except in the case of slender delta wings. A representation close to that of Goman-Khrabrov can however be used, the aerodynamic coefficient is written as :  $C = f(\alpha) + g(\alpha)F + C_{\dot{\alpha}} \dot{\alpha}$  with  $\tau_1 \dot{F} + F = F_0(\alpha - \tau_2 \dot{\alpha})$  where  $F$  is an internal variable representative of the effect of vortical lift ( $F=0$  corresponds to the absence of vortex burst effect i.e. the vortex bursts upstream of the wing apex,  $F=1$  corresponds to the maximum vortical effect i.e.  $x \geq 1$ ). During a test with increasing angle of attack, if  $\dot{\alpha} > A$  the vortex burst phenomenon is delayed and thus  $x=1$  is obtained for any angle of attack. In the same way, during tests with decreasing angle of attack, if  $\dot{\alpha} < -A$  the vortex burst point is maintained upstream of the wing apex. Moreover, making the assumption that the direct effect of  $\dot{\alpha}$  (term in  $C_{\dot{\alpha}} \dot{\alpha}$ ) is negligible in comparison with the vortical effects (this hypothesis seems to be true for CN), the term  $f(\alpha) + g(\alpha)$  is then given by

the test with increasing angle of attack and  $f(\alpha)$  by the test with decreasing incidence. Using the value of the time-constant  $\tau_1^* = 15$  given in [1], as well as the empirical formulas to obtain  $x_0(\alpha)$  from [3]:

$$x_0 = \frac{N}{\tan \gamma - B} - A \text{ where}$$

$$\gamma = \cos^{-1}(\cos \alpha \cos \Lambda)$$

$$N = 0.33 + \frac{0.3}{\tan^2 \Lambda}$$

$$A = 0.2 + \frac{1.6}{\tan^2 \Lambda}$$

$$B = 0.43 - \frac{0.1}{\tan \Lambda}$$

the validity of these assumptions for tests on the "PQR" apparatus is shown (figure 5). By using the preceding results, the expression of the function F is deduced. F is plotted versus incidence, figure 6, for tests carried out at various values of  $\alpha$ . The results of simulations of the CN models with  $\tau_1^* = 10$  and  $\tau_2^* = 5$  are shown figure 7 for sinusoidal type tests of various amplitudes. The interesting capability of prediction of a model using only very few tests (tests at constant pitch rate for maximum, minimal and null value) will be noted. The deviations observed between model and test results at low angles of attack and in the range of high angles of attack are mainly due to the fact that the functions f and g were identified with the tests at maximum and minimum pitch rate, in the whole range of angle of attack. The same structure of model could be used for the coefficient  $C_m$ . But the direct effect of  $\dot{\alpha}$  can not be neglected (see figure 8) as compared to the unsteady effect. Unsteady effects caused by  $\dot{\alpha}$  on the wing appear, for tests with increasing angle of attack as a delay of the incidence where the  $C_m$  slope changes. The behaviour for tests with decreasing angle of attack is very different in the range of angle of attack close to the fore body static instability, showing that the model of  $C_m$  has to take into account a more complete vortical diagram. In this case, the tests at constant  $\dot{\alpha}$  which sweeps in the whole range of angle of attack do not contain all the information necessary to the knowledge of the  $C_m$  behaviour. It seems necessary to take into account the vortex phenomena on the whole aircraft by means of specific tests.

## 5. USE OF NEURAL NETWORK MODEL

The results obtained in the last paragraphs show some limits of an approach of modelling by traditional techniques (transfer function, or model of knowledge including a descriptive internal variable of the streamline flow). An interesting alternative to circumvent this problem is the use of neural network models. This type of models is very well suited to non linear dynamical systems. It is shown that any continuous function can be approximated to any desired accuracy, by a network of one hidden layer of sigmoid units and one layer of linear output unit. Moreover this method is very flexible to use, because no simplifying assumption is necessary to identify the parameters of the model. The relation connecting the input  $(e_i)_{i=1,n}$

to the output  $y_k$  of a neuron unit is written in the form:  $y_k = f_k(\sum_j w_{k,j} e_j + \theta_k)$  where f is related to an

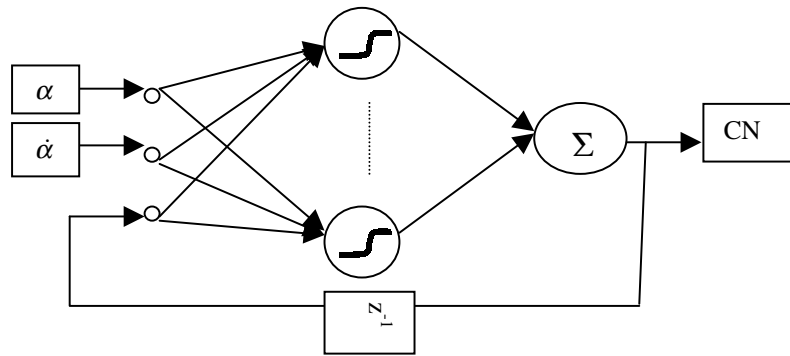
activation function of the neuron (in the case of a neuron with sigmoidal function of activation

$$f(x) = \frac{1}{1 + e^{-x}}).$$

For a network with several layers, the inputs of the layer k are the outputs of the k-1 layer. The parameters of the networks weight  $w_{k,j}$  and bias  $\theta_k$  are obtained by minimisation of a criterion of difference between output of the network and real output. The development of the neural model was carried out using the "neural network" MATLAB toolbox, the algorithm of minimisation uses the method of Levenberg-Marquardt. The criterion of minimisation is the sum of the squares of the difference between real outputs and neural network model model outputs.

### 5.1 CN MODEL

The structure of CN model defined previously, can be translated in a neural network with a hidden layer being composed of neurons with sigmoidal functions of activation.



This structure is of NNARX type, because it uses, for the calculation of  $CN(t)$ , only the inputs at the time  $t$  and the output  $CN$  at time  $t-\Delta T$ . The adjustment of the neural network was carried out by recreating a set of experiments with a simplified model of Goman-Khrabrov ( $CL_{\dot{\alpha}} = cste$  and

$$x_0(\alpha) = \frac{1}{2} \left( 1 - \tanh\left(\frac{\alpha - \alpha_0}{\Delta\alpha}\right) \right).$$

It was shown that a network containing between 30 and 60 neuron units gave very good results either in training or in generalisation (simulation of a test not belonging to the basis of training). The identification of the real model was made from tests on a generic configuration of delta wing plane with canard surface. The experimental base is composed of 50 tests on "PQR" apparatus : 10 tests at constant pitch rate where the angle of attack varies from  $40^\circ$  to  $90^\circ$  and 40 tests where the angle of attack varies in a sinusoidal way. A neural model comprising 60 neurons in the hidden layer, was identified by using half of the tests of the experimental data basis (other half being used to check the predictive capabilities of the model). The number of iterations of the optimisation process was voluntarily reduced for regularisation reasons. The choice of the tests to be incorporated in the basis of training was carried out by a trial and error process.

The results of the model in simulation are shown figure 9 (1 test out of 2 does belong to the basis of training). A simplified neural model (5 neurons in the hidden layer) trained with the tests at constant pitch rate was also identified. It could be mentioned (figure 10) that this model shows rather good agreements with tests except the tests of low amplitude.

## 5.2 Cm MODEL

There does not exist a formal model of the Goman-Khrabrov type making it possible to recreate a realistic set of experiments. The adjustments of the  $C_m$  model were thus copied on those of the model of  $CN$ . It was noticed that the addition of  $CN$  in the set of inputs of the network improved its performances. An element of explanation is that  $CN$  is a kind of memory of the vortex flow on the wing, the effect of delayed  $C_m$  would play a part to model the effects of the vortical field on the fore body. In order to obtain models with good performances, it was necessary to add to the basis of training, tests located in the zone of influence of the fore body vortex. Two thirds of the tests available were thus used. The results relative to this model are presented figure 11.

## 6. CONCLUSION

The interest of an approach aiming at including aerodynamic internal variables (effect of the vortical flow) in the model of behaviour of the longitudinal aerodynamic coefficients at high angles of attack has been shown. This approach made it possible to deliver a structure of representation of the coefficients close to the physics which can be translated in the form of a recurrent neural network model. These models were then applied on a complete aircraft configuration and have shown good performances.

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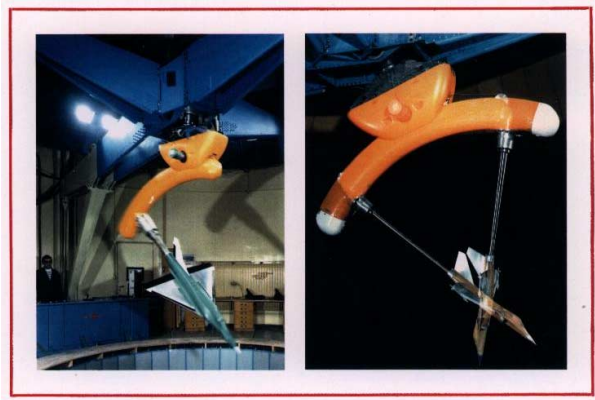


figure 1 : Rotary balance

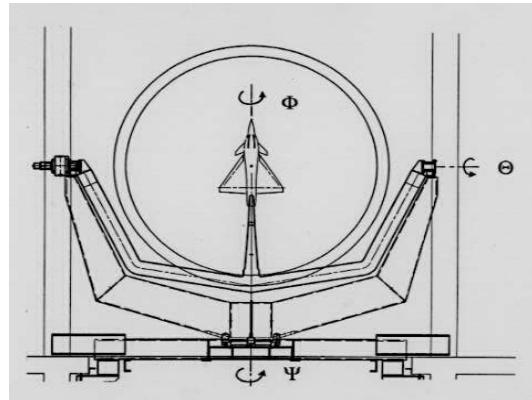


figure 2 : "PQR" apparatus

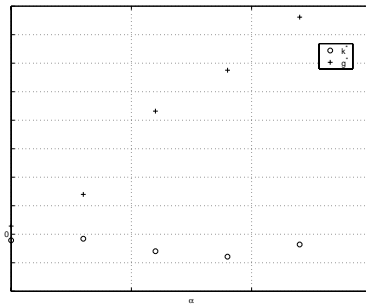


figure 3 : gain of the transfert function

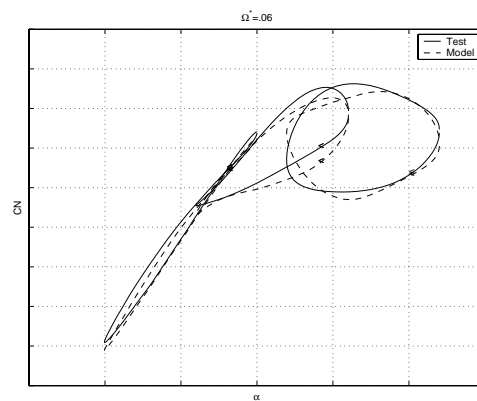
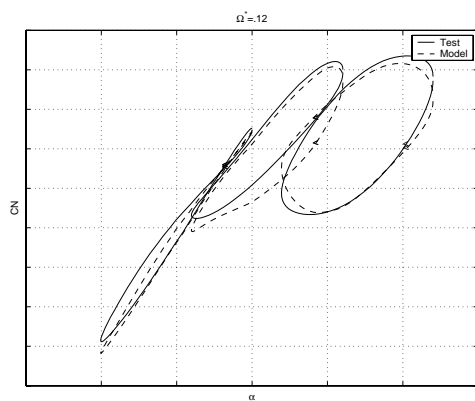
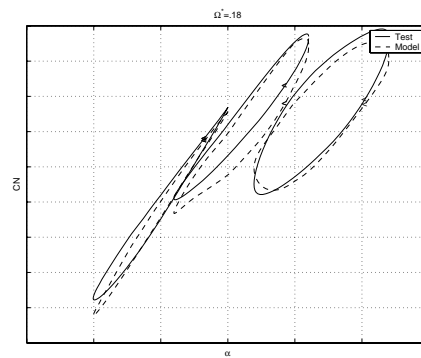


figure 4 : Comparison between test and model (transfert function model)



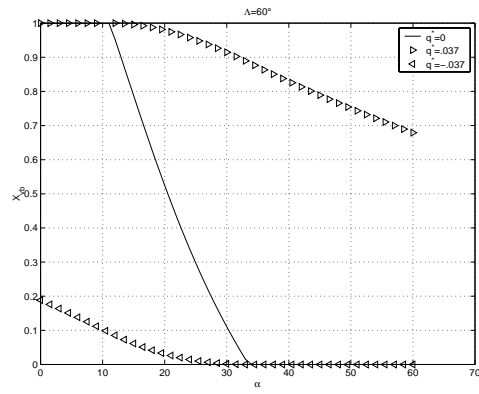


figure 5 : simulation of vortex burst position

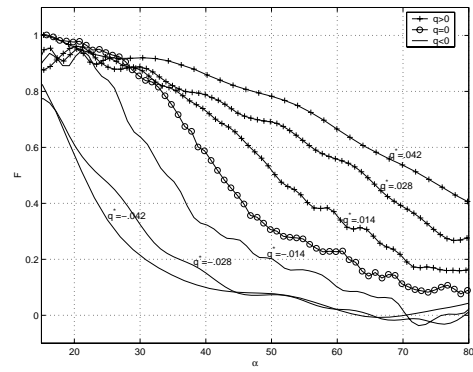


figure 6 : Evolution of F

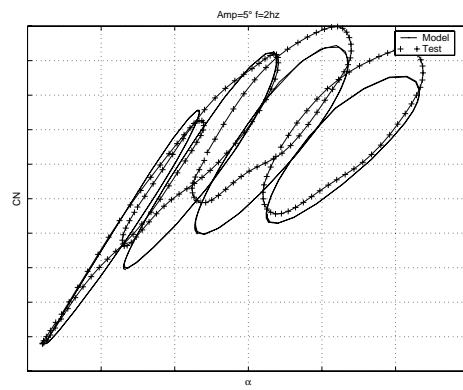
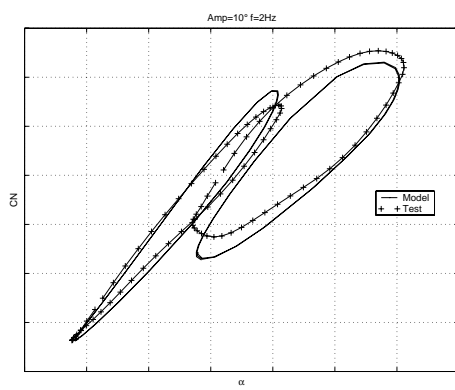


figure 7 : Comparison between test and model (simplified « knowledge model »)

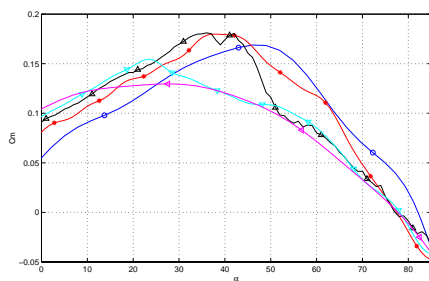
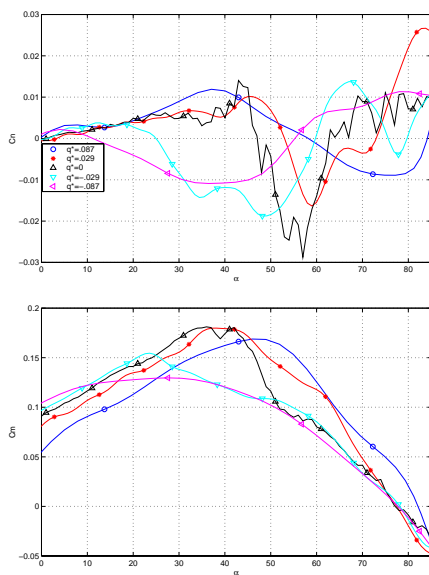


figure 8 : Cm,Cn vs  $\alpha$  at different q rate

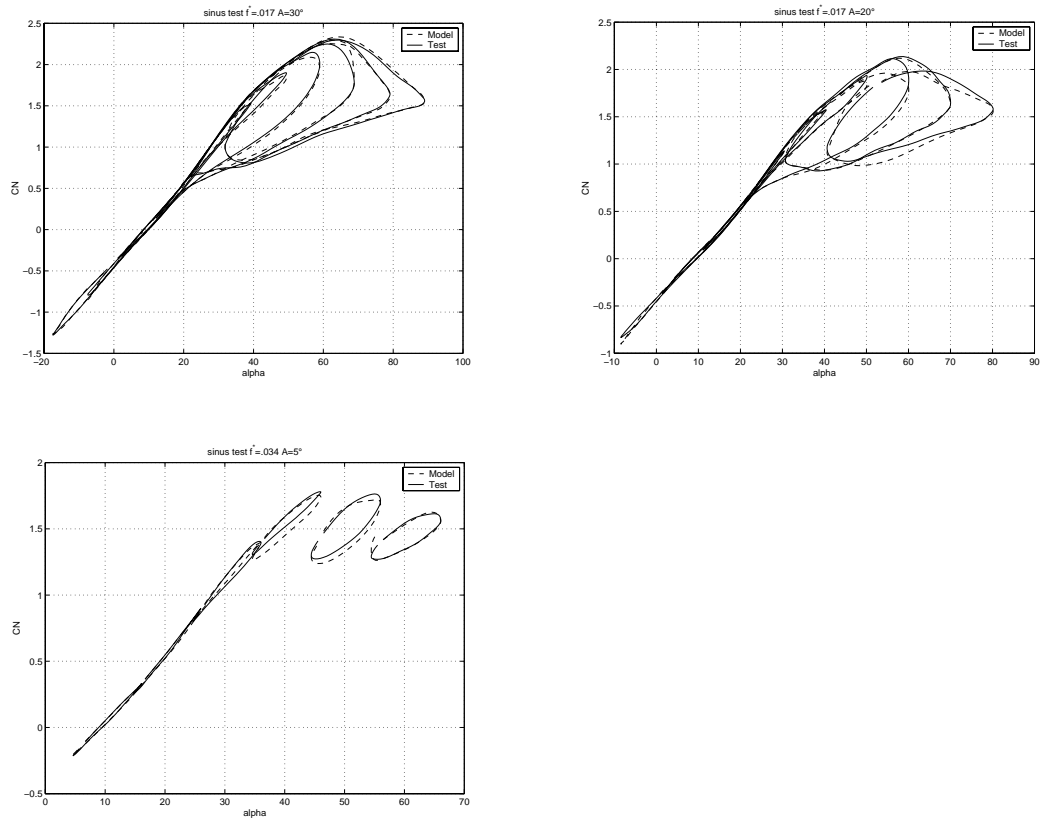


figure 9 : Comparison between test and model ( neural network model : 60 neurons)

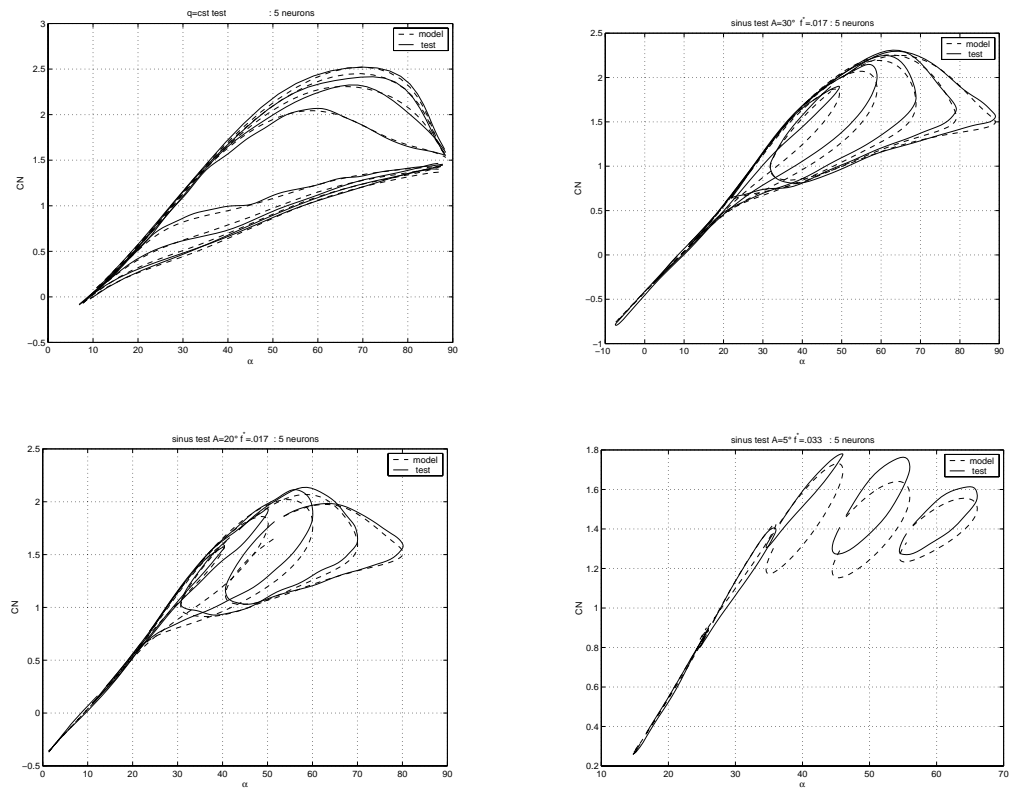


figure 10 : Comparison between test and model ( neural network model : 5 neurons)

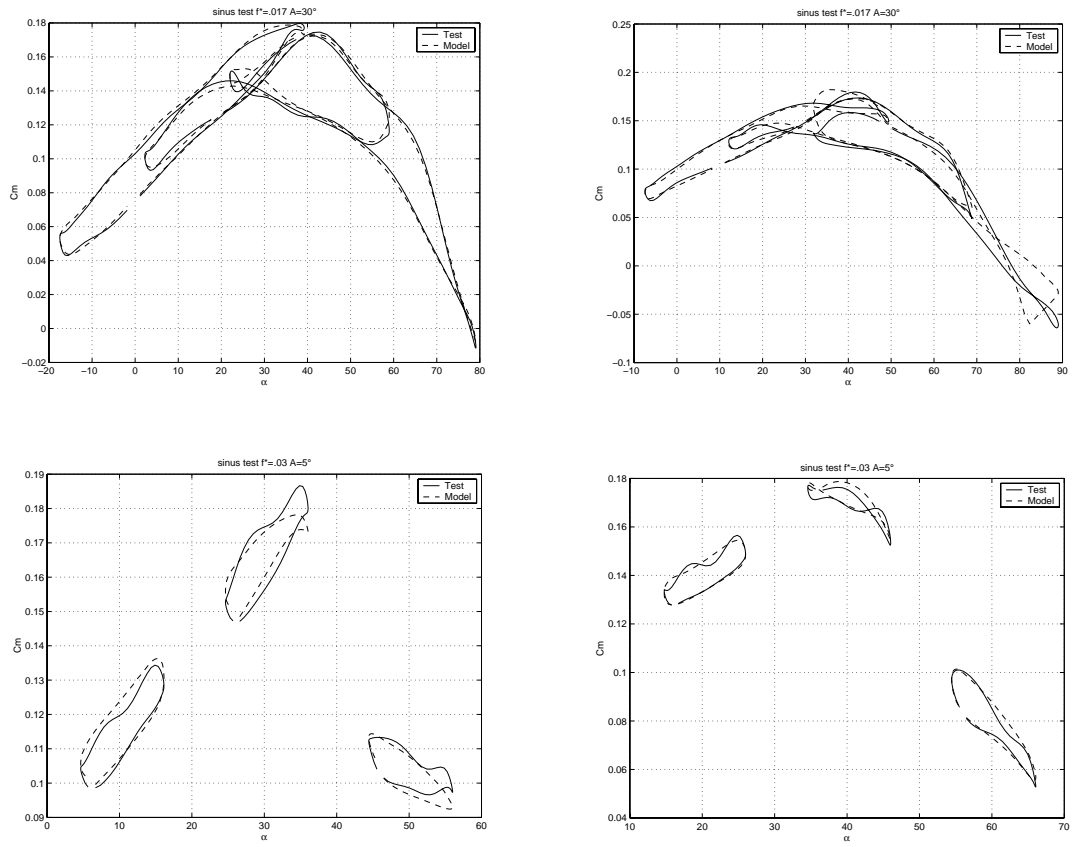


figure 11 : Comparison between test and model ( neural network model of  $C_m$  : 60 neurons)

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**Author: Mr. Plankaert**

**Question by Mr. Khrabroy:** Have you tried to use a similar approach to model more complex motion (instead of pure pitch motion). For example: oscillatory coning motions, motions with  $\beta$  variation?

**Answer:** Not yet, but if this approach works for the pitch motion (model with recurrent neural network), we would try to apply this model to data obtained from our rotary balance. (This is much more complicated because we have to take into account motion in the longitudinal and lateral direction of the vortex and because there are some difficulties to get rid the influence of the dynamic rig on the measurement at different mean angles of attack).

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